
Appendix 1: Function atan2

The ratio of two real numbers defines a single-valued function of two arguments:

$$\text{atan2}(y,x) = \begin{cases} \arctan(y/x), & \text{for } x > 0 \\ \pi + \arctan(y/x), & \text{for } y \geq 0, x < 0 \\ -\pi + \arctan(y/x), & \text{for } y < 0, x < 0 \\ \pi/2, & \text{for } y > 0, x = 0 \\ -\pi/2, & \text{for } y < 0, x = 0 \\ \text{undefined}, & \text{for } y = 0, x = 0 \end{cases}$$

Here, it is applied for calculation of a single value of angle φ given through $\sin \varphi$ and $\cos \varphi$.

Example A1.1:

(a) Equation $\sin \varphi = 1/2$ has two solutions

$$\varphi = \begin{cases} \arcsin(1/2) \\ \pi - \arcsin(1/2) \end{cases} = \begin{cases} \pi/6 \\ 5\pi/6 \end{cases}$$

Equation $\cos \varphi = \sqrt{3}/2$ also has two solutions

$$\varphi = \begin{cases} \arccos(\sqrt{3}/2) \\ -\arccos(\sqrt{3}/2) \end{cases} = \begin{cases} \pi/6 \\ -\pi/6 \end{cases}$$

(b) Two simultaneous equation $y \equiv \sin \varphi = 1/2$ and $x \equiv \cos \varphi = \sqrt{3}/2$ results in a single value. Namely, since $x = \sqrt{3}/2 > 0$, the first line in the atan2 definition gives

$$\text{atan2}(y,x) = \arctan(y/x) = \arctan \frac{1/2}{\sqrt{3}/2} = \pi/6.$$

Appendix 2: Synthesis of the FSF using Mathematica[®] software system

A2.1 Basic HTM matrices of the FSF theory

A2.1.1 Matrices of the elementary motions

In the following Mathematica[®] code the 4×4 HTM matrices of elementary motions (Table 3.5) are labelled $m_1, m_2, m_3, m_4, m_5,$ and m_6 ,

$$\begin{aligned}
 m1 &= \{ \{1, 0, 0, \mathbf{xx}\}, \{0, 1, 0, 0\}, \{0, 0, 1, 0\}, \{0, 0, 0, 1\} \} \\
 m2 &= \{ \{1, 0, 0, 0\}, \{0, 1, 0, \mathbf{yy}\}, \{0, 0, 1, 0\}, \{0, 0, 0, 1\} \} \\
 m3 &= \{ \{1, 0, 0, 0\}, \{0, 1, 0, 0\}, \{0, 0, 1, \mathbf{zz}\}, \{0, 0, 0, 1\} \} \\
 m4 &= \{ \{1, 0, 0, 0\}, \{0, \text{Cos}[\text{th}], -\text{Sin}[\text{th}], 0\}, \{0, \text{Sin}[\text{th}], \\
 &\text{Cos}[\text{th}], 0\}, \{0, 0, 0, 1\} \} \\
 m5 &= \{ \{ \text{Cos}[\text{ps}], 0, \text{Sin}[\text{ps}], 0 \}, \{0, 1, 0, 0\}, \{ -\text{Sin}[\text{ps}], 0, \\
 &\text{Cos}[\text{ps}], 0 \}, \{0, 0, 0, 1\} \} \\
 m6 &= \{ \{ \text{Cos}[\text{ph}], -\text{Sin}[\text{ph}], 0, 0 \}, \{ \text{Sin}[\text{ph}], \text{Cos}[\text{ph}], 0, 0 \}, \\
 &\{0, 0, 1, 0\}, \{0, 0, 0, 1\} \}
 \end{aligned} \tag{A2.1}$$

A2.1.2 The error matrix

In the following Mathematica[®] code the 4×4 HTM error matrix, Eq. (8.9), are labelled **de1**,

$$\begin{aligned}
 \text{de1} &= \{ \{0, -\text{dph}, \text{dps}, \text{dx}\}, \{ \text{dph}, 0, -\text{dth}, \text{dy} \}, \\
 &\{ -\text{dps}, \text{dth}, 0, \text{dz} \}, \{0, 0, 0, 0\} \}
 \end{aligned} \tag{A2.2}$$

For convenience, some error matrices associated with FSS links are prepared. If, for example, the number of the form-shaping system links is less than 11 (including the fixed link No. 0), the following 10 matrices are defined:

$$\begin{aligned}
 \text{de10} &= \{ \{0, -\text{dph0}, \text{dps0}, \text{dx0}\}, \{ \text{dph0}, 0, -\text{dth0}, \text{dy0} \}, \\
 &\{ -\text{dps0}, \text{dth0}, 0, \text{dz0} \}, \{0, 0, 0, 0\} \} \\
 \text{de11} &= \{ \{0, -\text{dph1}, \text{dps1}, \text{dx1}\}, \{ \text{dph1}, 0, -\text{dth1}, \text{dy1} \}, \\
 &\{ -\text{dps1}, \text{dth1}, 0, \text{dz1} \}, \{0, 0, 0, 0\} \} \\
 \text{de12} &= \{ \{0, -\text{dph2}, \text{dps2}, \text{dx2}\}, \{ \text{dph2}, 0, -\text{dth2}, \text{dy2} \}, \\
 &\{ -\text{dps2}, \text{dth2}, 0, \text{dz2} \}, \{0, 0, 0, 0\} \} \\
 &\dots \\
 \text{de19} &= \{ \{0, -\text{dph9}, \text{dps9}, \text{dx9}\}, \{ \text{dph9}, 0, -\text{dth9}, \text{dy9} \}, \\
 &\{ -\text{dps9}, \text{dth9}, 0, \text{dz9} \}, \{0, 0, 0, 0\} \}
 \end{aligned} \tag{A2.3}$$

A2.1.3 Saving main matrices

To append the listed matrix definitions to any developed file, the entered symbols are saved using the command

```
Save["matr", {m1, m2, m3, m4, m5, m6, del10, del11, del12, del13,
del14, del15, del16, del17, del18, del19}]
```

 (A2.4)

where "matr" and "del" are the filenames.

To manipulate with the matrices using their symbols in any processed file, the name "matr" is entered as follows:

```
<<matr
```

 (A2.5)

where command "<<" allows reading the symbols in Eq. (A4) and evaluation of all read files within the processed file.

A2.1.4 Basic HTM matrices for the planar (2-D) case

- Three matrices of the elementary motions associated with symbols mx , my , mz are defined:

```
mx={{1, 0, xx}, {0, 1, 0}, {0, 0, 1}}
my={{1, 0, 0}, {0, 1, yy}, {0, 0, 1}}
mph={{Cos[ph], Sin[ph], 0}, {-Sin[ph], Cos[ph], 0}, {0, 0, 1}}
```

 (A2.6)

- The error matrix, Eq. (8.9), is defined:

```
eps ={{0, -dph, dx}, {dph, 0, dy}, {0, 0, 0}}
```

 (A2.7)

For convenience, four error matrices associated with FSS links are prepared,

```
eps0={{0, -dph0, dx0}, {dph0, 0, dy0}, {0, 0, 0}}
eps1={{0, -dph1, dx1}, {dph1, 0, dy1}, {0, 0, 0}}
eps2={{0, -dph2, dx2}, {dph2, 0, dy2}, {0, 0, 0}}
eps3={{0, -dph3, dx3}, {dph3, 0, dy3}, {0, 0, 0}}
```

To append the listed matrix definitions to a developed file, the entered symbols are saved:

```
Save["matr3", {mx, my, mph, eps0, eps1, eps2, eps3}]
```

 (A2.8)

where "matr3" is the filename.

To manipulate with the matrices using only their symbols in any processed file, Eq. (A2.8), is entered as <<matr3.

A2.1.5 Transformation of the HTM matrices and vectors to the third order form

The 4×4 matrices $m4$, $m5$, $m6$ (matrices of rotations) and the 4×1 vectors can be transformed into the regular matrices and vectors of the third order using the 3×4 matrix

$$T_{4to3} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

In terms of Mathematica® codes,

$$\mathbf{T4to3} = \{ \{1, 0, 0, 0\}, \{0, 1, 0, 0\}, \{0, 0, 1, 0\} \} \quad (\text{A2.9})$$

Examples:

(a) The code for matrix transformation

```
In[1] := m6 = {{Cos[ph], -Sin[ph], 0, 0}, {Sin[ph], Cos
             [ph], 0, 0}, {0, 0, 1, 0}}, {0, 0, 0, 1}};
In[2] := T4to3.m6.Transpose[T4to3]
Out[2] := {{Cos[ph], -Sin[ph], 0}, {Sin[ph], Cos[ph], 0}, {0, 0, 1}}
```

(b) The code for vector transformation

```
In[1] := T4to3.{x,y,z,1}
Out[1] := {x,y,z}
```

where the point "." stands for matrix multiplication; $\text{In}[n] :=$ is the input information; and $\text{Out}[n] :=$ is the n th output.

A2.2 Examples: Solutions of problems considered in this book

A2.2.1 Synthesis of manipulation matrix, Eq. (4.38)

Given: Coordinate code of the lathe is $k = 631$. **Required:** Synthesis of the manipulation matrix.

Solution:

- The code starts from the command `<<matr`, Eq. (A2.5), to add matrices of links motion m_6 , m_3 , and m_1 , Eq. (A2.1) to the processed file. The identifiers A_{01} , A_{12} , A_{23} are assigned to matrices m_6 , m_3 , and m_1 , respectively;
- Manipulation matrix A_{03} is obtained by the ordered multiplication $A_{01}.A_{12}.A_{23}$;
- The result is shown in the matrix form.

```
In[1] := <<matr
        "Links motion matrices";
        A01=m6;
        A12=m3;
        A23=m1;
        "Manipulation matrix";
        A03=A01.A12.A23;
        "A03 ="MatrixForm[A03]
```

Out [n] :=

$$\mathbf{A03} = \begin{bmatrix} \cos ph & -\sin ph & 0 & xx \cos ph \\ \sin ph & \cos ph & 0 & -xx \sin ph \\ 0 & 0 & 1 & zz \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

A2.2.2 Synthesis of the form-shaping function, Eq. (4.41)

Given: Coordinate code of the lathe is $k = 631$. **Required:** Synthesis of the form-shaping function (FSF). **Solution:**

- The code starts from seven commands of the *Example A2.1*;
- The single-point tool vector $\mathbf{r}_3 = [0, 0, 0, 1]^T$ is entered in the form $\mathbf{r3} = \{0, 0, 0, 1\}$ and multiplied by the manipulation matrix.
- The Output demonstrates the FSF in the matrix form.

```
In [1] := <<matr;
          "Links motion matrices";
          A01 = m6;
          A12 = m3;
          A23 = m1;
          "Manipulation matrix";
          A03 = A01.A12.A23;
          "Single point-tool vector";
          r3={0,0,0,1};
          r0=A03.r3;
          "r0 ="MatrixForm[r0]
```

$$\text{Out [n] := } \mathbf{r0} = \begin{bmatrix} xx \cos ph \\ -xx \sin ph \\ zz \\ 1 \end{bmatrix} \quad (\text{A2.10})$$

A2.2.3 Setup (table 6.4)

Given: FSF of the lathe from the previous Example. On the lathe, the cylinder of radius R is machined using longitudinal turning. **Required:** (a) Equation of the machined cylinder; (b) equation of the unit vector of the normal to the machined cylinder; (c) graphical presentation of the cylinder of radius $R = 50$ mm and length 120 mm, the initial cross section is located on $z = 20$ mm. **Solution:**

(a) *Input description:*

- The code starts from vector $\mathbf{r0}$, Eq. (A2.10);
- The single constraints $xx = R$ is applied using replacement operator $/$. $xx \rightarrow R$, and the HTM vector is rearranged to a Cartesian coordinate vector using matrix T_{4to3} , Eq. (A2.9), with renaming $\mathbf{r0}$ for \mathbf{rcyl} ;

- Partial derivatives dph with respect to ph and dz with respect to zz are defined using operator $D[rcyl, ph]$ and $D[rcyl, zz]$, respectively;
- The normal NN is defined through vector cross product $Cross[dph, dz]$;
- Unit normal vector nn is defined and simplified through equation $nn = NN/(NN.NN)^{1/2}$;
- The cylinder $rcyl50$ of radius 50 is defined using replacement operator $/.$ $R \rightarrow 50$;
- Using commands $ParametricPlot3D[rcyl50, \{ph, 0, 2\ Pi\}, \{zz, 20, 140\}]$, the cylinder plot is built. The command allows building the surfaces given in the two-parametric form, $rcyl = rcyl(ph, zz)$. The parameter limits are $0 < ph \leq 2\pi$ and $20 < zz \leq 140$

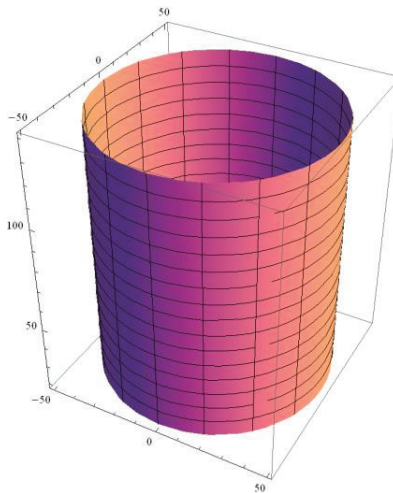
(b) *Output description:*

Formula of the unit vector of the normal gives two possible values, $nn = \pm[R \cos ph, R \sin ph, 0]^T$;

- The plot shows the cylindrical surface between $z = 20$ and $z = 140$.

```
In[n] := <<matr;
r0={xx Cos[ph], xx Sin[ph], zz, 1};
rcyl=Tr4to3.r0 /. xx -> R
dph=D[rcyl, ph];
dz=D[rcyl, zz];
NN=Cross[dph, dz];
nn=Simplify[NN/(NN.NN)^(1/2)]
rcyl50=rcyl /. R->50;
ParametricPlot3D[rcyl50, {ph, 0, 2 Pi}, {zz, 0, 100}]
```

$$\text{Out}[n] := \left\{ -\frac{R \cos ph}{\sqrt{R^2}}, \frac{R \sin ph}{\sqrt{R^2}}, 0 \right\}$$



The conventional coordinate frame used in Mathematica differs from the frame accepted in the lathe (see system $X_1 Y_1 Z_1$ in Figure 4.7). To transform the coordinate systems, two successive rotations are entered in the previous code: around the Y -axis at 90° and around the Z -axis at -90° . See the following code, where the output is related to the lathe accepted frame:

```

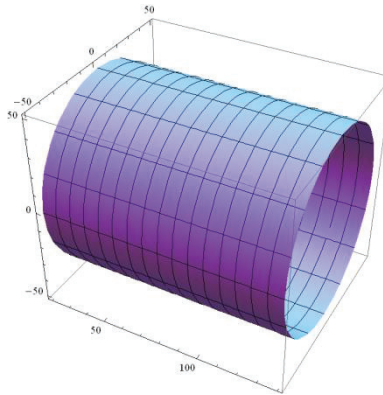
In[n] := <<matr;
r0={xx Cos[ph], -xx Sin[ph], zz, 1};
rL=(m5/.ps->Pi/2). (m6/.ph->-Pi/2). r0
rcyl=Tr4to3.rL/.xx->R;
dph=D[rcyl, ph];
dc=D[rcyl, zz];
NN=Cross[dph, dc];
Nn=Simplify[NN/(NN.NN)^(1/2)]
Rcyl=rcyl/.R->50;
ParametricPlot3D[Rcyl, {ph, 0, 2 Pi}, {zz, 20, 140}]

```

```

Out[n] := {0, - $\frac{R \cos ph}{\sqrt{R^2}}$ ,  $\frac{R \sin ph}{\sqrt{R^2}}$ }

```



A2.2.4 Geometrical accuracy of the 2-DOF (RR) manipulator, Eq. (10.1-6)

Given: Dimensions of the 2-DOF manipulator (Fig. 10.1): the end-effector (EE) point is given through position vector $\mathbf{r}_0 = [0, 0, 1]^T$; $a_1 = a_2 = 250$ mm; geometrical errors are $\delta_{x0} = \delta_{y0} = 0$; $\delta_{\varphi0} = 0$; $\delta_{x1} = \delta_{y1} = 0.01$ mm; $\delta_{\varphi1} = 0.0005$; $\delta_{x2} = \delta_{y2} = 0.1$ mm; $\delta_{\varphi2} = 0$; $\delta_{x3} = \delta_{y3} = 0.01$ mm; $\delta_{\varphi3} = 0.0005$; $\delta_{x4} = \delta_{y4} = 0.1$ mm; and $\delta_{\varphi4} = 0$. **Required:** The distance error δ_{AB} between the two position of the EE point: configuration A ($\varphi_{2A} = \pi/12$; and $\varphi_{4A} = \pi/6$) and configuration B ($\varphi_{2B} = \pi/6$; and $\varphi_{4B} = \pi/3$).

Solution: The In[n] commands are entered in the given order:

- The code fetches file matr3, Eq. (A2.9), with the list of 3×3 matrices;

The motion matrices A01 with new parameter pfi1, A12 with new parameter a2, A23 with new parameter pfi2, and A34 with new parameter a4;

- The manipulating matrix 0A_4 , Eq. (10.1), called A04;
- The reciprocal matrices A10(-ph1), A21(-a2), A32(-ph3), and A43(-a4);
- The total error matrix E, Eq. (10.3), called "Et";
- The numerical data condA, condB, and condE;
- The position errors e_{rA} of point A and e_{rB} of point B;
- The distance error d_{AB} is defined.

The Out[n] consists of a single number for verification of the code.

```
In [n] := <<matr3;
"FSF manipulating matrix";
A01=mph/.ph→phi1;
A12=mx/.xx→a2;
A23=mph/.ph→phi3;
A34=mx/.xx→a4;
A04=Simplify[A01.A12.A23.A34];eps0
A10=mph/.ph→-phi1;
A21=mx/.xx→-a2;
A23=mph/.ph→-phi3;
A34=mx/.xx→-a4;
r4={0,0,1};
r0=A04.r4;
"Error matrix";
Et = Simplify[eps0+A01.eps1.A10+A01.A12.eps2.A21.A10 +
A01.A12.A23.eps3.A32.A21.A10 +
A01.A12.A23.A34.eps4.A43.A32.A21.A10];
"Numerical Example";
condA={phi1→Pi/12, phi3→Pi/6, a2→250, a4→250};
condB={phi1→Pi/6, phi3→Pi/3, a2→250, a4→250};
condE={dx0→0, dy0→0, dph0→0,
dx1→0.01, dy1→0.01, dph1→0.0005,
dx2→0.1, dy2→0.1, dph2→0,
dx3→0.01, dy3→0.01, dph3→0.0005,
dx4→0.1, dy4→0.1, dph4→0};
"Two points";
erA=N[(Et.r0)/.condA/.condE];
erB=N[(Et.r0)/.condB/.condE];
"Distance AB";
dAB=((erA-erB).(erA-erB))^(1/2)
```

```
Out [n] := 0.333215 mm
```

A2.2.5 Inverse kinematics of the GSP platform, Eq. (5.38-40)

Given: The form-shaping system combines the base (coordinate frame S_0) and the moving platform (coordinate frame S_1); dimensions of the GSP platform (Fig. 5.26): $r_1 = 700$ mm; $r_2 = 250$ mm; angles of the limb ends location on the base plane are α_i ($i = 1, 2, \dots, 6$); angles of the limb ends location on the moving platform plane are β_i ($i = 1, 2, \dots, 6$). **Required:** (a) Lengths of the limbs in the general case; (b) Lengths of the limbs of the 3–3 platform in the configuration $x = y = 0$; $z = 400$ mm; $\theta = \psi = \varphi = 0$, with end limbs location $\alpha_1 = \alpha_6 = -\pi/3$; $\alpha_2 = \alpha_3 = \pi/3$; $\alpha_4 = \alpha_5 = \pi$; $\beta_1 = \beta_2 = 0$; $\beta_3 = \beta_4 = 2\pi/3$; $\beta_5 = \beta_6 = 4\pi/3$.

Solution:

```

In[n] := <<matr;
  "FSF";
  A01 = m1.m2.m3.m4.m5.m6;
  "Lower ends of limbs in coordinate frame S0";
  rM1={r1 Cos[a11],r1 Sin[a11],0,1};
  rM2={r1 Cos[a12],r1 Sin[a12],0,1};
  rM3={r1 Cos[a13],r1 Sin[a13],0,1};
  rM4={r1 Cos[a14],r1 Sin[a14],0,1};
  rM5={r1 Cos[a15],r1 Sin[a15],0,1};
  rM6={r1 Cos[a16],r1 Sin[a16],0,1};
  "Upper ends of limbs in coordinate frame S1";
  rQ1={r2 Cos[bet1],r2 Sin[bet1],0,1};
  rQ2={r2 Cos[bet2],r2 Sin[bet2],0,1};
  rQ3={r2 Cos[bet3],r2 Sin[bet3],0,1};
  rQ4={r2 Cos[bet4],r2 Sin[bet4],0,1};
  rQ5={r2 Cos[bet5],r2 Sin[bet5],0,1};
  rQ6={r2 Cos[bet6],r2 Sin[bet6],0,1};
  "Upper ends of limbs in coordinate frame S0";
  rU1= A01.rQ1;
  rU2= A01.rQ2;
  rU3= A01.rQ3;
  rU4= A01.rQ4;
  rU5= A01.rQ5;
  rU6= A01.rQ6;
  "Limb lengths in the general case";
  L1=((rU1-rM1).(rU1-rM1))^(1/2);
  L2=((rU2-rM2).(rU2-rM2))^(1/2);
  L3=((rU3-rM3).(rU3-rM3))^(1/2);
  L4=((rU4-rM4).(rU4-rM4))^(1/2);
  L5=((rU5-rM5).(rU5-rM5))^(1/2);
  L6=((rU6-rM6).(rU6-rM6))^(1/2);
  "Numerical example: 3-3 GSP"
  cond={r1→700, r2→250,
    al1→ -Pi/3, al2→ Pi/3, al3→ Pi/3, al4→ Pi, al5→ Pi,
    al6→ -Pi/3,
    bet1→ 0, bet2→ 0, bet3→ 2Pi/3, bet4→ 2Pi/3, bet5→
    4Pi/3, bet6→ 4Pi/3, xx→0,yy→0,zz→400,th→0,ps→0,ph→0};
  "Limb lengths"
  N[L1/.cond]
  N[L2/.cond]
  N[L3/.cond]
  N[L4/.cond]
  N[L5/.cond]
  N[L6/.cond]

```

```
Out[n] := Limb lengths
733.144
733.144
733.144
733.144
733.144
733.144
```

A2.2.6 Error matrix E and Jacobian J_c for 2RR robot

In the very beginning, the code fetches file “matr3” with the list of 3×3 matrices. Then, the following subjects are computed in the given order:

- The manipulating matrix 0A_4 , Eq. (10.1), called “A04”;
- The total error matrix E, Eq. (10.149), called “Et”;
- Twist δ , Eq. 10.154), called “tw”; and,
- The Jacobian matrix J, Eq. (10.157), called “Jt”.

Remember that the command **Simplify**[*qqq*] simplifies of the bracketed expression *qqq*, the command **Transpose**[*mmm*] transposes the bracketed matrix *mmm*, and the command **Inverse**[*mmm*] carries out inversion of the square matrix *mmm*.

```
In[1] := "3RR robot";
<<matr3;
"FSF manipulating matrix";
A01=mph /. phi->phi1;
A12=mx/.xx->a2;
A23=mph /. phi->phi3;
A34=mx/.xx->a4;
A04=Simplify[A01.A12.A23.A34];
"FSF variation";
Et= Simplify[eps0+A01.eps1.Inverse[A01]+
      A01.A12.eps2.Inverse[A01.A12]+A01.A12.A23.eps3.
      Inverse[A01.A12.A23]+A01.A12.A23.A34.eps4.
      Inverse[A01.A12.A23.A34]];
"Twist";
tw={Et[[1,4]],Et[[2,4]],Et[[2,1]]};
"Jacobian";
Jt=Transpose[{D[tw,dx0],D[tw,dy0],D[tw,dph0],
D[tw,dx1],D[tw,dy1],D[tw,dph1],D[tw,dx2],D[tw,dy2],
D[tw,dph2],D[tw,dx3],D[tw,dy3],D[tw,dph3],
D[tw,dx4],D[tw,dy4],D[tw,dph4]}];
```

A2.2.7 Diagnostic procedure: Computation of Eq. (10.174)

```
In[1] := "Structural matrix";
js={{Cos[0], Sin[0],0,0,0,1},
```

```

{Cos[Pi/3], Sin[Pi/3], 0, 0, 0, 1},
{Cos[2Pi/3], Sin[2Pi/3], 0, 0, 0, 1},
{Cos[3Pi/3], Sin[3Pi/3], 0, 0, 0, 1},
{Cos[4Pi/3], Sin[4Pi/3], 0, 0, 0, 1},
{Cos[5Pi/3], Sin[5Pi/3], 0, 0, 0, 1},
{Cos[0], Sin[0], -100 Sin[0], 100 Cos[0], 100, 1},
{Cos[Pi/3], Sin[Pi/3], 100 Sin[Pi/3], 100
  Cos[Pi/3], 100, 1},
{Cos[2Pi/3], Sin[2Pi/3], -100 Sin[2Pi/3], 100
  Sin[2Pi/3], 100, 1},
{Cos[3Pi/3], Sin[3Pi/3], -100 Sin[3Pi/3], 100
  Cos[3Pi/3], 100, 1},
{Cos[4Pi/3], Sin[4Pi/3], -100 Sin[4Pi/3], 100
  Cos[4Pi/3], 100, 1},
{Cos[5Pi/3], Sin[5Pi/3], -100 Sin[5Pi/3], 100
  Sin[5Pi/3], 100, 1}};

"Results of measurements, Eq. (10.172)";
meas={0.01, 0.02, 0.02, 0.03, 0.02, 0.01, 0.02, 0.03, 0.03,
0.03, 0.02, 0.02};

"Estimates Eq. (10.174)"
Inverse[N[Transpose[js]].js].N[Transpose[js]].meas

```

```

Out[n] := {-0.0057578, .00288675, -0.0000310551, -0.000002773,
0.00006667, 0.0183333}

```

A2.2.8 The Fichter's singular configuration of the 3-3 GSP matrix (Fig. 11.13)

The Jacobian matrix for Fichter's singular configuration is built. A part of the code is identical to that in Appendix A.2.2.5. The demonstrated results are the determinant graph of the Jacobian matrix vs. angle φ (see Out[65]) and zero value of the determinant for $\varphi = \pi/2$ (see Out[66]).

```

In[1] := "Fichter singularity of the 3-3 GPS";

```

```

<<matr;
  A01=m1.m2.m3.m4.m5.m6;

"Lower ends of limbs";

rM1={r1 Cos[a11], r1 Sin[a11], 0, 1};
rM2={r1 Cos[a12], r1 Sin[a12], 0, 1};
rM3={r1 Cos[a13], r1 Sin[a13], 0, 1};
rM4={r1 Cos[a14], r1 Sin[a14], 0, 1};
rM5={r1 Cos[a15], r1 Sin[a15], 0, 1};

```

```
rM6={r1 Cos[a16],r1 Sin[a16],0,1};
```

```
"Upper ends of limbs in S1";
```

```
rQ1={r2 Cos[bet1],r2 Sin[bet1],0,1};
```

```
rQ2={r2 Cos[bet2],r2 Sin[bet2],0,1};
```

```
rQ3={r2 Cos[bet3],r2 Sin[bet3],0,1};
```

```
rQ4={r2 Cos[bet4],r2 Sin[bet4],0,1};
```

```
rQ5={r2 Cos[bet5],r2 Sin[bet5],0,1};
```

```
rQ6={r2 Cos[bet6],r2 Sin[bet6],0,1};
```

```
"Upper ends of limbs in S0";
```

```
rU1=A01.rQ1;
```

```
rU2=A01.rQ2;
```

```
rU3=A01.rQ3;
```

```
rU4=A01.rQ4;
```

```
rU5=A01.rQ5;
```

```
rU6=A01.rQ6;
```

```
"Limb vectors";
```

```
l1=rU1-rM1;
```

```
l2=rU2-rM2;
```

```
l3=rU3-rM3;
```

```
l4=rU4-rM4;
```

```
l5=rU5-rM5;
```

```
l6=rU6-rM6;
```

```
"Limb lengths";
```

```
L1=(l1.l1)^(1/2);
```

```
L2=(l2.l2)^(1/2);
```

```
L3=(l3.l3)^(1/2);
```

```
L4=(l4.l4)^(1/2);
```

```
L5=(l5.l5)^(1/2);
```

```
L6=(l6.l6)^(1/2);
```

```
"Limb orts";
```

```
e1=l1/L1;
```

```
e2=l2/L2;
```

```
e3=l3/L3;
```

```
e4=l4/L4;
```

```
e5=l5/L5;
```

```
e6=l6/L6;
```

```
"Limb arms";
```

```
arm1=A01.rQ1/.zz-> 0;
```

```
arm2=A01.rQ2/.zz-> 0;
```

```

arm3=A01.rQ3/.zz-> 0;
arm4=A01.rQ4/.zz-> 0;
arm5=A01.rQ5/.zz-> 0;
arm6=A01.rQ6/.zz-> 0;

```

```

"Ort moments";
n1=Cross[T4to3.arm1, T4to3.e1];
n2=Cross[T4to3.arm2, T4to3.e2];
n3=Cross[T4to3.arm3, T4to3.e3];
n4=Cross[T4to3.arm4, T4to3.e4];
n5=Cross[T4to3.arm5, T4to3.e5];
n6=Cross[T4to3.arm6, T4to3.e6];

```

```

"Jacobian matrix synthesis"

```

```

JJ={ {e1[[1]],e1[[2]],e1[[3]],n1[[1]],n1[[2]],n1[[3]]},
      {e2[[1]],e2[[2]],e2[[3]],n2[[1]],n2[[2]],n2[[3]]},
      {e3[[1]],e3[[2]],e3[[3]],n3[[1]],n3[[2]],n3[[3]]},
      {e4[[1]],e4[[2]],e4[[3]],n4[[1]],n4[[2]],n4[[3]]},
      {e5[[1]],e5[[2]],e5[[3]],n5[[1]],n5[[2]],n5[[3]]},
      {e6[[1]],e6[[2]],e6[[3]],n6[[1]],n6[[2]],n6[[3]]} };

```

```

cond = {r1-> 0.7, r2-> 0.25, a11-> -Pi/3, a12-> Pi/3, a13->
Pi/3, a14-> Pi, a15-> Pi, a16-> -Pi/3, bet1-> 0, bet2-> 0,
bet3-> 2Pi/3, bet4-> 2Pi/3, bet5-> 4Pi/3, bet6-> 4Pi/3, xx->
0, yy-> 0, zz-> 0.4, th-> 0, ps-> 0};

```

```

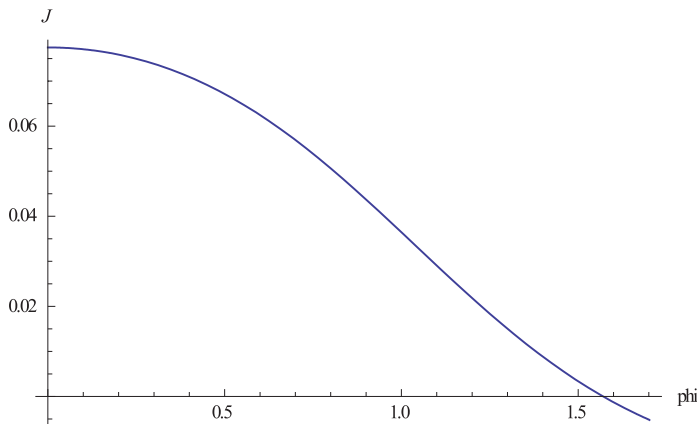
JJphi=JJ/.cond;
detJ=Det[JJphi];
Plot[detJ, {ph, 0, Pi/2+Pi/24}, AxesLabel->{phi, J}]

```

```

Out[65] :=

```



```

In[66] := Chop[N[detJ/.ph-> Pi/2]]
Out[66] := 0

```

Appendix 3: Some symbols and operations from the Mathematica® software system

A3.1 Global objects

A3.1.1 Input and output

In[n] is the n th input information entered by the user;

Out[n] is an output resulting from code processing generated by the software system.

A3.1.2 Algebraic symbols

Regular symbols “+”, “-”, “×”, “/”, and “()” are used for algebraic operations on scalar values and functions; “=” is the assignment symbol;

The power symbol is “^”, i.e., x^y means x^y ;

Dot “.” is the symbol of product of matrices, matrices by vectors, and the dot-product (scalar product) of two vectors.

Attention:

- instead of product symbol “×”, the blank is regularly used for multiplication of scalars, i.e., $a \times b \equiv a b$;
- square brackets are used for giving parameter/s of commands and functions;
- braces are used a list of elements, vectors and matrices.

A3.2 Matrix and Vector Synthesis

- *Vectors of the third order*

```
In[n] := r = {x, y, z}
```

```
Out[n] := {x, y, z}
```

where “=” is the assignment symbol.

- *Position vector of the fourth order*

```
In[n] := r = {x, y, z, 1}
```

```
Out[n] := {x, y, z, 1}
```

- *Non-position vector of the forth order*

```
In[n] := r = {x, y, z, 0}
```

```
Out[n] := {x, y, z, 0}
```

- *Matrix of the 4×4 order (an expression in the braces is the matrix row)*

```
In[n] := A = {{a11, a12, a13, a14}, {a11, a12, a13, a14},  
             {a11, a12, a13, a14}, {a11, a12, a13, a14}}
```

```
Out[n] := {{a11, a12, a13, a14}, {a11, a12, a13, a14}, {a11,  
           a12, a13, a14}, {a11, a12, a13, a14}}
```

A3.3 Operations on Vectors and Matrices

- *Sum of the third order vectors*

```
In[n] :=      r1 = {x1, y1, z1};
              r2 = {x2, y2, z2};
              rs = r1 + r2
Out[n] :=      {x1 + x2, y1 + y2, z1 + z2}
```

- *Sum of position and non-position vectors*

```
In[n] :=      r1 = {x1, y1, z1, 1}
              r2 = {x2, y2, z2, 0}
              rs = r1 + r2
Out[n] :=      {x1 + x2, y1 + y2, z1 + z2, 1}
```

- *Sum of two non-position vectors*

```
In[n] :=      r1 = {x1, y1, z1, 0}
              r2 = {x2, y2, z2, 0}
              rs = r1 + r2
Out[n] :=      {x1 + x2, y1 + y2, z1 + z2, 0}
```

- *Sum of the matrices (e.g., of the 4×4 order)*

```
In[n] :=      A = {{a11, a12, a13, a14}, {a21, a22, a23, a24},
                  {a31, a32, a33, a34}, {a41, a42, a43, a44}};
              B = {{b11, b12, b13, b14}, {b21, b22, b23, b24},
                  {b31, b32, b33, b34}, {b41, b42, b43, b44}};
              AS = A + B
Out[n] :=      {{a11 + b11, a12 + b12, a13 + b13, a14 + b14}, {a21
+ b21, a22 + b22, a23 + b23, a24 + b24}, {a31 +
b31, a32 + b32, a33 + b33, a34 + b34}, {a41 + b41,
a42 + b42, a43 + b43, a44 + b44}}
```

- *Product of two real or numbers*

```
In[n] :=      x = a;
              y = b;
              x y
Out[n] :=      a b
```

- *Dot product of vectors*

```
In[n] :=      r1 = {x1, y1, z1};
              r2 = {x2, y2, z2};
              rp = r1.r2
Out[n] :=      x1 x2 + y1 y2 + z1 z2
```

- *Vector (cross) product of two vectors of the third order*

```
In[n]:=      r1 = {x1, y1, z1};
              r2 = {x2, y2, z2};
              rp = Cross[r1,r2]
Out[n]:=     {-y2 z1 + y1 z2, x2 z1 - x1 z2, - x2 y1 + x1 y2}
```

- *Product of the matrix and vector*

```
In[n]:=      A = {{a11, a12, a13, a14}, {a21, a22, a23, a24},
                  {a31, a32, a33, a34}, {a41, a42, a43, a44}};
              r1 = {x, y, z, v};
              r = A.r1
Out[n]:=     {a14 v + a11 x + a12 y + a13 z, a24 v + a21 x + a22
              y + a23 z, a34 v + a31 x + a32 y + a33 z, a44 v + a4
              1x + a42 y + a43 z}
```

- *Product of two matrices (e.g., the 3×4 matrix by 4×2 matrix)*

```
In[n]:=      A={{a11,a12,a13,a14},{a21,a22,a23,a24},{a31,a32,a33,
                  a34}};
              B={{b11,b12},{b21,b22},{b31,b32},{b41,b42}};
              A.B
Out[n]:=     {{a11 b11 + a12 b21 + a13 b31 + a14 b41,
              a11 b12 + a12 b22 + a13 b32 + a14 b42},
              {a21 b11 + a22 b21 + a23 b31 + a24 b41,
              a21 b12 + a22 b22 + a23 b32 + a24 b42},
              {a31 b11 + a32 b21 + a33 b31 + a34 b41,
              a31 b12 + a32 b22 + a33 b32 + a34 b42}}
```

- *Demonstration*

- Symbol *expr*; evaluates *expr*, but ignores demonstration of the result in the Output.
- Symbol “text” ignores evaluation of the text and demonstrates it as is.
- Command `MatrixForm[matr]` demonstrates matrix *matr* in the regular matrix form:

```
In[n]:=      matr={{a11, a12}, {a21, a22}};
              MatrixForm[matr]
Out[n]:=     
$$\begin{bmatrix} a11 & a12 \\ a21 & a22 \end{bmatrix}.$$

```


Abbreviations

AE – active element of the FSS

CMM – coordinate measuring machine

CNC – computer numerical control

DC – dimensional chain

DOF – degree of freedom

EE – end-effector

FSF – form-shaping function

FSS – form-shaping system

GSP – Gough-Stewart platform

HTM – homogeneous transformation matrix

IR – industrial robot

IT – international tolerance

LMS – least-mean square

LVDT – linear variable differential transformer

MinZ – minimum zone

MT – machine tool

NC – numerical control

PWM – precision working machine

PKM – parallel-kinematics machine

SKM – serial-kinematics machine

TE – target element of the FSS

WM – working machine

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