APPLICATIONS OF MATHEMATICAL HEAT TRANSFER AND FLUID FLOW MODELS IN ENGINEERING AND MEDICINE

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APPLICATIONS OF MATHEMATICAL HEAT TRANSFER AND FLUID FLOW MODELS IN ENGINEERING AND MEDICINE

Abram S. Dorfman

University of Michigan, USA

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In memory of my favorite co-author,
who passed away too soon,
Professor of University of Toledo,
Ella Fridman.

Contents

Series Preface

The *Wiley-ASME Press Series in Mechanical Engineering* brings together two established leaders in mechanical engineering publishing to deliver high-quality, peer-reviewed books covering topics of current interest to engineers and researchers worldwide. The series publishes across the breadth of mechanical engineering, comprising research, design and development, and manufacturing. It includes monographs, references, and course texts. Prospective topics include emerging and advanced technologies in engineering design, computer-aided design, energy conversion and resources, heat transfer, manufacturing and processing, systems and devices, renewable energy, robotics, and biotechnology.

Applications of Mathematical Heat Transfer and Fluid Flow Models in Engineering and Medicine

No problem can be solved from the same level of consciousness that created it.

Albert Einstein

Preface

This textbook for advanced graduate and post-graduate courses presents the applications of the modern heat transfer and fluid flow mathematical models in engineering, biology, and medicine. By writing this work, the author continues the introduction of brand-new efficient methods in fluid flow and heat transfer that have been developed and widely used during the last 50 years after computers became common. While his previous two monographs presented these contemporary methods on an academic level in heat transfer only [119] or in both areas heat transfer and fluid flow on the preliminary level [121], this manual introduces the modern approaches in studying corresponding mathematical models—a core of each research means determining its efficiency and applicability. Two types of new mathematical models are considered: the conjugate models in heat transfer and in fluid flow and models of direct numerical simulation of turbulence. The current situation of applications of these models is presented in two parts: applications of conjugate heat transfer in engineering (Part I) and applications of conjugate fluid flow (peristaltic flow) in medicine and biology and applications in engineering of direct numerical simulation (Part II). These parts contain theory, analysis of mathematical models, and methods of problem solution introduced via 134 detailed and 231 shortly reviewed examples selected from a list of 448 cited original papers adopted from 152 scientific general mathematic, computing, and different specific oriented journals, 42 proceedings, reports, theses, and 37 books. This list of 448 references comprehends the whole period of the new methods existing from the 1950s to present time, including more than 100 results published during the last 5 years among which more than half of the studies were issued in 2014 to 2016.

The term conjugate, or coupled problem, was coined in the 1960s to designate the heat transfer strict investigation that requires matching temperature fields of bodies flowing around or inside the fluids. Later on, it became clear that these terms and procedures are important to many other natural and technology processes, consisting of interactions between elements and/or substances. In particular, the peristaltic flow is an inherent conjugate phenomenon because such flow occurs due to interaction between the elastic channel walls and the fluid inside channel. The conjugate formulation reflects the basic features of a studied phenomenon. Due to that, the models of this type are reliable and significantly improve the correctness and physical understanding of the results. Conjugate methods constitute a powerful tool for solving contemporary problems, substituting the previous approximate approaches. At the same time, it is important to know when the common more simple approaches may be used with comparable exactness, instead of more complex conjugate procedures. The textbook answers this question, as well as significant other questions governing the applications of conjugate methods.

The other group of new methods considered in this text is based on direct numerical solution of exact unsteady (without averaging) Navier-Stokes equations. Because the unsteady Navier-Stokes equations describe the complete space- and time-dependent field of turbulent flow, the results of direct numerical simulation are considered as an experimental data gained computationally. Such results provide highly accurate instantaneous turbulence characteristics giving further insight into physics of turbulence, opening new possibilities, fresh ideas and improving applications.

The discussion goes along with 239 exercises and 136 comments. Whereas the former allowed the reader to improve his or her skills and experience, the latter are used to clear up specific terms and to note some instructive historical facts. The majority of exercises are used by the author to divide the derivation of particular expressions or formulae with a reader. To realize such an offer, the way of solution and the result are given in the text. However, the mathematical procedure is left for the reader as an exercise. Such a type of exercise gives a person a choice to be satisfied only by results, or use the suggested drill to improve their own expertise. For convenience, it is pointed out in the text when each exercise should be performed, and to find a specific one, the reader may apply the contents where the locations of exercises are indicated. Also, for the reader orientation, the more sophisticated exercises and examples (and hence, corresponding publications) are marked by an asterisk (*).

As mentioned above, comments provide significant information required for understanding. Such valuable subjects as, for example, special means, like tridiagonal matrix algorithm (TDMA), or alternating direction implicit method (ADI), or scientific terms, such as order of the value of magnitude, function singularities, tensor or factor of nonisothermicity, are explained via comments incorporated in the text. Meaningful historical notes are also introduced through comments at the relevant manual points. Thus, after discussing the benefits of the boundary layer theory, it is noticed that boundary-layer methods was not utilized for the first 25 years until Prandtl's lecture at the Royal Aeronautical Society meeting in 1927. The other examples of historical notes given by comments are explanations of the name BBO of differential equation, the Saffman slip boundary condition, the Paul Erlich role in monoclonal antibodies, and the Smagorinsky contribution in the direct numerical simulation of turbulence.

In view of the intended audience, special attention is given to the balance between strictness and comprehensibility of the writing. Such a compromise is realized using a strict formulation of the problems on one side and the detailed explanation of definitions, special terms and procedures on the other. For example, it is justified that both problems—heat transfer of flow past a body and peristaltic flow in a flexible channel—are similar, and both are inherently conjugate. At the same time, it is explained in detail why a nonlinear model of peristaltic flow differs in essence from a linear heat transfer pattern.

In contrast to exciting college courses on heat transfer presenting basically simple empirical approaches based on the heat transfer coefficients, the conjugate methods are grounded on contemporary fluid flow and heat transfer models. Therefore, to help the reader to understand the conjugate principles and procedures, the third part of this textbook offers fundamental laws of laminar and turbulent fluid flows and applied mathematic methods frequently used in engineering (Part III). Setting subsidiary chapters behind the body text, it is assumed that the reader takes a relatively small part of the information required to understand only a specific thesis or topic, rather than studying the whole subject in advance. In addition, the references given in the text in the form: Chap. (Chapter), S. (Section), Exam. (Example), Exer. (Exercise), and Com. (Comment) help the reader to find directly the desired portion of knowledge. Such a book structure permits the reader to get explanation step by step during studying. At the same time, an experienced person may read the text ignoring those citations.

As a whole, the textbook is written so as to be usable to senior and post-graduate students and engineers with the prerequisites of calculus, fluid mechanics, and heat transfer college engineering courses.

The textbook begins with Part I presenting applications in heat transfer, which starts with an introduction containing two pieces. The first writing, "When and why conjugate procedure is essential" explains in detail where the term conjugate came from, what it means, and in which cases conjugation procedure is important. The second piece entitled "A core of conjugation" presents the qualitative analysis of a simple problem of heat transfer from a plate heated from one end. This assay clarifies a physical meaning of the conjugation principle by showing the contrasting distributions of the heat transfer characteristics on the interface in two flow directions, from heated and unheated ends.

This part consists of four chapters, incorporating the theory of conjugate heat transfer based on universal functions (Chapter 1) and three chapters of applications: universal function applications (Chapter 2), conjugate problem applications in flows around bodies and inside channels (Chapter 3), and special application of conjugate heat transfer models in industrial and technological processes (Chapter 4).

The first chapter begins from the formulation of conjugate heat transfer problems specifying two sets of equations, the initial and boundary conditions governing the conjugate problem for a body and fluid. Each equation, such as Navier-Stokes or Laplace equation, is followed by references to chapter or section from the third part, presenting appropriate explanation. The initial conditions, the three kinds of common boundary conditions, and the Dirichlet and Neumann problem formulation for elliptic differential equations are considered in detail. The conjugate conditions on fluid/body interface (fourth kind boundary condition) and specific methods for performing the conjugate procedure are discussed also.

The next section introduces the universal functions that are widely used in this text. It explains what universal functions are, and shows that these types of functions are proper and convenient tools for nonisothermal and conjugate heat transfer analyzing. Two forms of universal functions, integral and differential, are employed.

First, the special form of Duhamel's integral containing influence function is derived which, in fact, presents a universal function for the heat transfer on a plate with arbitrary temperature and zero pressure gradient flow.

Then, the equivalent differential universal function, in the form of a series of temperature head derivatives, is obtained by the consecutive differentiation of the Duhamel's integral. The calculation data for the series coefficients and for the exponents of influence function in Duhamel's integral conclude the determination of the universal functions for laminar flows.

Because the universal functions are valid in the same form for other regimes and conditions, the remaining part of the first chapter specifies only the series coefficients and the appropriate exponents of the influence function for differential and integral forms of universal functions. These results are obtained for the following cases: turbulent flow, compressible zero pressure gradient flow, power-law Non-Newtonian fluids, moving through surrounding continuous sheet, plate with unsteady arbitrary temperature distribution, flow past axisymmetric body, inverse universal functions for arbitrary heat flux distribution, and functions for recovery factor.

Chapter 2 provides applications of universal functions. General properties of conjugate heat transfer are investigated, considering the conjugate problem as a case of heat transfer from arbitrary nonisothermal surface. The results are obtained analyzing universal functions and are supplemented with relevant examples. It is found that: (i) the second term of series with the first derivative in differential universal function basically determines the effect of the temperature head gradient because the first coefficient of series is from 3 to 10 times larger than the second one, whereas the others are negligible small, (ii) because the first coefficient is positive, the increasing temperature heads (positive derivative) leads to greater and the decreasing temperature heads (negative derivative) results in lesser heat transfer coefficients than that for an isothermal surface, (iii) strikingly large effects, resulting in zero heat transfer if the negative derivative is large or the surface is long enough, (iv) the positive and negative pressure gradients respectively decrease and increase the heat transfer coefficient of nonisothermal surface, (v) the higher the Prandtl number is, the less the effect of nonisothermicity in turbulent flow is, and the higher the Prandtl and Reynolds numbers, the less the effect of nonisothermicity in turbulent flow is, (vi) the effect of nonisotermicity caused by variable time temperature is greater than that of variable space temperature, and (vii) the Biot number specifies the degree of problem conjugation and shows that in both limiting cases, $\mathrm{Bi} \to \infty$ and $\mathrm{Bi} \to 0$ conjugate problem decays, so that the greatest effect of conjugation occurs at comparable body/fluid resistances at $\mathrm{Bi} \approx 1$.

The second part of Chapter 2 describes some inherent characteristics and phenomena for conjugate heat transfer, indicating that:

- The differential universal function builds up the general convective boundary condition testifying that a series with only the first term constructs the boundary condition of the third kind, taking into account only isothermal effect, whereas retaining of the followed terms increases the accuracy of boundary conditions, accounting for the effect of the first and higher temperature head derivatives.
- Because the second term with the first derivative basically defines the effect of nonisothermicity, the calculation of its value gives an estimation of error caused by a boundary condition of the third kind telling us whether the conjugate solution is required or the simple common approach is acceptable.

- Using a general boundary condition allows reducing the conjugate problem to equivalent conduction problem for the body only.
- There exists a gradient analogy, which means that the temperature head gradient affects the heat transfer coefficient, as the free stream velocity gradient influences the friction coefficient.
- In the case of decreasing temperature head, the heat flux inversion might occur when the heat flux vanishes—a phenomenon analogous to separation of boundary layer in flows with adverse pressure gradient.

Chapter 3 presents results of conjugate heat transfer investigations in flows around bodies (external flows) and inside the channels and tubes (internal flows) in general, without specifying a concrete application in any device or process. The examples reviewed in this chapter differ by methods of problem solution, form of objects, boundary conditions, flow regimes, and state of flow (initial or developed). These results present effects of different parameters and conditions on conjugate heat transfer intensity in external and internal flows in general, without reference to particular application. The specific practical applications of conjugate heat transfer are discussed in the next chapter.

Special attention is given to conjugate heat transfer in flows past the thin plate, which are considered first. Due to the relative simplicity of this type of problem, we used the universal functions to create effective methods and obtain significant results, which include: (i) investigation of the temperature singularities on the solid/fluid interface, (ii) creation of the charts for simple conjugate problems solution, (iii) consideration of examples to help a reader to possess the charts usage, and (vi) computation of the inequalities for quasi-steady approach validation.

The other part of this chapter contains 15 reviewed and 27 indicated as other works of original studies of conjugate heat transfer in external and internal flows. Here, as well as in the following Chapters 4 and 6, the original studies are presented describing problem formulation, a mathematical model as the system of equations, ideas of solutions, and the basic results, but without exercises, which would be difficult for beginners.

The following examples are reviewed:

- Past plate and bodies: laminar flow past finite rectangular slab, flush sources on an infinite slab, free convection on vertical and horizontal thin plates, elliptic cylinder in laminar flow, translating fluid sphere, radiating plate with internal source, and radiating thin plate in compressible flow.
- Inside channels and tubes: fully developed laminar flow in a pipe heated by uniform heat flux, turbulent flow in parallel plats duct at periodical inlet temperature, fully developed flow in thick-walled channel with moving wall, hydrodynamically and thermally developed flow in a thick-walled pipe, laminar flow in the entrance of plane duct, flow in a channel of finite length, unsteady heat transfer in a duct with laminar flow, and transient heat transfer in a pipe with constant surface temperature.

Chapter 4 contains specific conjugate heat transfer applications in different industrial areas and technology processes. Thirty-one original papers are reviewed in four sections considering heat exchangers and finned surfaces (12 examples), thermal treatment and cooling systems (9), simulation of industrial (3) and technological (7) processes. Chapter 4 begins with conjugate solution of the classical problem of overall heat transfer coefficient of two flows separated by a thin plate, which is usually considered as a model of heat exchanger. Six conjugate solutions of this problem using different methods are analyzed, showing how much the conjugate strict results differ from data obtained by simple common approach. The following solutions are considered: two solutions of concurrent and countercurrent laminar and turbulent flows, solutions for two quiescent and two flowing fluids separated by vertical plate, and vertical thin-walled pipe with forced inside and natural outside flows.

The conjugate results for overall heat transfer coefficient obtained for a thin plate are compared with exact two-dimensional conjugate solution to understand where assumption of thin plate is applicable, and how otherwise such results should be corrected. It is found that conjugate results for thin plate are practically accurate, except a small area close to the leading edge, where two-dimensional effects are important and should be taken into account. The next three examples present more reliable heat exchanger models: two conjugate models using double pipes and a special model for the microchannel exchanger. The two last samples of this section consider models of finned surfaces.

In the second section of Chapter 4, the thermal treatment of moving continuous materials is analyzed in the first three examples, and conjugate heat transfer in different cooling systems is studied in the other six examples. The heat transfer in electronic packages is discussed in the first two examples, the results for cooling turbine blades and vanes are presented in the next two examples, and the last two samples analyzed the protection of systems in reentry rocket, and in a nuclear reactor at emergency loss of coolant. The next section gives three examples of simulation of the processes in industrial equipment. Because of complexity, there are relatively few publications of this type. The three models considered here simulate processes in twin-screw extruder, optical fiber coating, and continuous wires casting.

The last section of Chapter 4 presents heat transfer investigations in seven technological processes. The first three examples examine heat and mass transfer in multiphase flows using models of such complicated processes; wetted-wall absorber, concrete production, and Czochralski crystal growing. The next three samples studied drying of wood board, porous materials, and pulled through coolant continuous sheets. The last example presents freeze drying of two specimens of food. Forty-seven relevant other works are introduced shortly after the reviewed examples.

Part I is closed by a short summary of results. The basic dependences of heat transfer characteristics are presented in the form of a table where the influence parameters are arranged in order of a degree of conjugation. Such comparative information is useful in making a decision whether the conjugate solution is needed in a particular problem, or the common simple approach is enough to solve it. This question is discussed in detail and possible recommendations are formulated.

Part II incorporates two chapters consisting of applications of modern methods in fluid flow. The theory and general characteristics of those methods in both areas, peristaltic flow and direct turbulence simulation, are outlined in Chapter 5. Applications of peristaltic flow in medicine, biology and engineering and applications of direct simulation of turbulence in engineering are introduced in Chapter 6, reviewing 24 and presenting 42 as other works of original papers.

Chapter 5 starts from considering the peristaltic motion as conjugate phenomenon. Physical analysis shows that peristaltic motion adopted from creation exists due to conjugation (say interaction) between flexible walls and fluid inside tubular human organs, so that the conjugation nature is an inherent property of peristaltic flow. These considerations are confirmed by subsequent examples of human organs operating under the peristaltic flow and by explaining working principles of some devices simulating this natural motion mechanism.

The next part of Chapter 5 consists of the formulation of the conjugate model for peristaltic motion. This model is similar to that for heat transfer described in detail in Part I, and involves two subdomains with conjugate conditions on the wall/fluids interface. Conjugate relations in the case of peristaltic flow contain no-slip conditions for velocities and the balance of forces on the interface instead of equalities of temperatures and heat fluxes in the case of heat transfer. The essential difference between both conjugate models is explained, stressing that nonlinear peristaltic flow model is more complex than the linear heat transfer one. To take into account that complexity after analyzing published studies, the term "semi-conjugate model" is introduced which describes the situation when only the effect of flexible walls on the flow is investigated, ignoring more complicated impacts of flow on walls motion. The samples reviewed in Chapter 6 show that the majority of studies are of the semi-conjugate type.

The discussion of the problem solutions begins from the first simple research. The main objective of early studies was the understanding of the peristalsis mechanism in order to get insight into physical processes in the ureter, like reflux of bacteria. To simplify the problems, early authors used a linear model and assumptions of low Reynolds number and long wavelength, which are often applied up to now. Two more substantial nonlinear semi-conjugate solutions are introduced next: the analytical solution at low Reynolds number based on a perturbation series and numerical solution of a two-dimensional peristaltic flow at a moderate Reynolds number.

Two examples are analyzed to introduce fully conjugate solutions, taking into account both effects of interaction of the flexible walls and fluid inside channel. In the early paper, the equality conditions of forces on the interface are defined, employing the relatively simple approximate expressions. Conjugate conditions on the interface in another study published later are much more complicated but more realistic. These conditions are constructed using relations from the theory of thin oscillating elastic plate and two-dimensional Navier-Stokes equations and result in a system of three differential equations. Both solutions are compared with corresponding semi-conjugate data showing that the flow significantly affects the wall's behavior.

The second part of Chapter 5 presents the modern methods of direct numerical simulation of turbulence. Here, the discussion starts with a short introduction explaining the difficulties associated with extremely wide scales of turbulence eddies, which range from scales of integral length to Taylor and Kolmogorov smallest scales. In the following three sections, the new methods: direct numerical simulation (DNS), large eddy simulation (LES) and detached eddy simulation (DES) are introduced and compared.

A direct numerical simulation is a method to solve the unsteady Navier-Stokes equations in order to obtain the complete space- and time-dependent field of turbulent flow. By estimation of a number of grid points and time steps required for performing DNS, it is shown that only relatively simple engineering problems at real Reynolds numbers can be investigated by direct simulation.

A large eddy simulation is a method of reduction of the requirements for DNS in order to solve directly Navier-Stokes equations at higher Reynolds numbers. The main idea of LES proposed by Smagorinsky is to separate the treatment of large and small eddies, computing the large eddies by DNS and small eddies by Reynolds-average models. To demonstrate how the filtering procedure works providing the separation of areas with DNS and Reynolds-average models, a simple filter based on the integration is described.

The filtered form of Navier-Stokes equations are analyzed showing that this procedure gives the field of filtered large scales modified by the subgrid scales stresses (SGS). This pattern represents the interaction between large and small eddies testifying the essential role of modeling SGS in LES.

Large eddy simulation significantly widened the application of the direct solutions of the Navier-Stokes equation. However, the important engineering applications such as airfoil, ground or marine vehicle require much higher Reynolds numbers and, accordingly, greater numbers of grid points and time steps. These large requirements are caused by the near-wall region with the smallest eddies whose role increases about three times proportional to the value of Reynolds number. To reduce the number of grid points and to achieve further progress, Spalart, with co-authors, suggested detached eddy simulation. This method (DES) is a hybrid approach combining the RANS (Reynolds-average Navier-Stokes equation) for near-wall region and the LES for domain with large eddies. To provide the model behavior according to required treatment by LES or by RANS, the blending functions are used. The idea of a blending function is described showing the principle of comparing the closest to the surface distance d with the largest grid sell Δ so that the model uses RANS close to the walls, where $d << \Delta$, and works as a subgrid type pattern away from walls, where $\Delta << d$.

Some examples demonstrate the accomplishments of DES in modeling the flow separations at high Reynolds numbers, such as sub- and super-critical flows around sphere and flows past aircraft models. Nevertheless, to correct weaknesses of DES, two modifications were proposed: the delayed detached eddy simulation (DDES) and the zonal detached eddy simulation (ZDES). In these versions of DES, the treatment of the area where the model switches from RANS to LES is improved in order to get rid of the rapid decrease of the RANS eddy viscosity, which might result in strong instabilities. In DDES, to prevent this undesired depletion of the RANS strength, the switch into LES is delayed. In ZDES, this problem is resolved by introducing separated zones for RANS and LES where the regime in each zone is selected individually in line with requirements.

At the end of this chapter, a small paragraph represents the chaos theory, which studies phenomena sensitive to initial conditions, like weather, when the small variations in one location may result in widely different outcomes far away (butterfly effect). Though currently the chaos theory is not a tool for turbulence modeling, some characteristics of turbulence are of a chaotic kind, which gives hope of using the chaos theory in the future.

Chapter 6 represents applications of advanced peristaltic and turbulence models in biology, medicine and engineering. Examples of original studies are reviewed, as well as the heat transfer articles in Chapter 4, presenting problem formulation, mathematical models as systems of equations, ideas of solutions and basic results. The applications in biology and medicine are described in three sections analyzing blood flows in normal and pathologic vessels, flows in disordered human organs and biological transport processes. The first section presents flow in the arterial stenosis and flow through series of stenoses, blood flow affected by magnetic field during MRA and MRI tests, and blood flow under the hyperthermia cancer treatment. In the second section, the abnormal flows and/or irregular situations are simulated: the particle motion in ureter modeling of a bacterium or stone motility, chyme flow during gastrointestinal endoscopy and bile flow in a duct with stones. The third section describes fluid transport in the cerebral perivascular space, macromolecules transport in tumors, embryo transport modeling, and the bioheat transfer in human tissues. Twenty articles are indicated as other works.

The second part of Chapter 6 consists of applications in engineering contributed by peristaltic flow simulation (PFS), DNS, LES, and DES (4 and 9 reviewed examples of peristaltic flow and turbulence simulations, respectively). Each section of reviewed articles is followed by other works citations. 13, 15, 12, and 14 studies of PFS, DNS, LES, and DES including IDDES and ZDES, respectively, are indicated. We begin the discussion with peristaltic flow applications in engineering, and then consider the engineering applications of direct numerical turbulence simulation. The contribution of peristaltic flow in engineering is presented by four recent results obtained during the last five years, including the effects of chemical reaction, a micropumping systems optimization, the method of valve-less microfluidic peristaltic pumping design, and the construction of biomimetic swallowing robot published in 2015. These examples demonstrate the effectiveness of mathematical models in peristaltic motion applications, which cardinally changes the methods of investigation in this area.

The review of DNS examples starts from simulation of turbulent boundary layer at a relatively high Reynolds number as $Re_\theta = 2560$ published four years ago. The next two studies introduce the effects of Reynolds and Prandtl numbers in turbulent heat transfer, and a more involved recent study of exothermic gas-phase reaction in a packed bed. The last example and 15 latest results, including three articles published in 2016 cited as other works, show progress in DNS.

The next three examples demonstrate the advances in LFS via simulation of vortex and pressure fluctuation in aerostatic bearings, effects of equivalence ratio fluctuations in combustion chamber of gas turbine, and the heat transfer in pebble bed of nuclear reactor at a high temperature. Ten other works published during the last few years and the two most recent studies appearing in 2016 show in addition to reviewed articles the current situation in LES.

The last three examples display the great progress in studying the real objects characteristics by DES. The first result published 13 years ago presents patterns of sub- and super-critical flows over sphere confirming the well-known experimental data of early (82°) and much later (120°) separations in the first and second cases, respectively. Two other samples show recent investigations of complicated natural prototypes: Reentry-F flight experiment and free-surface flow around a submerged submarine fairwater. Both studies are performed at real values of Reynolds number of order $\approx 10^7$ and the Mach number about 20 in the first study and the Froude number about 0.4 in the second one. Fourteen modern articles employed DES including the latest versions DDES and ZDES cited as other works present manifold achievements of contemporary methods of direct numerical simulation of turbulence in studying the complex engineering systems.

As mentioned above, Part III serves as a subsidiary intended to help a reader to find information during studying of the basic text. Three chapters containing fundamental laws and methods compose this part: laminar and turbulent fluid flows and heat transfer (Chapters 7 and 8) and basic analytical and numerical methods in applications (Chapter 9). Chapter 7 starts with discussing two similar mechanisms of momentum, energy, and mass transport described by conservation laws. Physically grounded analysis shows that structures of Navier-Stokes, energy, and mass transfer equations are similar consisting of two groups of terms responsible for the molecular and convective transport processes.

The next several sections present different forms and properties of Navier-Stokes equations. The vector, vorticity, stream function, and irrotational invisced forms as well as the form in Einstein notations are considered. The other often used notations, Kronecker delta and Levi-Civita index are also explained. Some basic exact solutions of Navier-Stokes and energy equations

(Stokes problems, flow and heat transfer in a channel and a tube, stagnation point flow, and heat transfer in Couette flow) are analyzed. The two cases of simplified Navier-Stokes equations, the small (creeping flows) and large (boundary layer) Reynolds numbers, are presented. As an example of creeping flow, the Stokes flow around a sphere is shortly described. The derivation of boundary layer equations and dimensionless numbers are given using the comparison of the terms order in Navier-Stokes and energy equations. The merits of boundary layer approach are described. The Prandtl-Mises and Görtler forms of boundary layer equation are analyzed. The physical meaning of several dimensionless numbers is explained indicating that each number may be interpreted as a ratio of particular physical parameters. As examples of exact solutions of boundary layer equations, the Blasius, Pohlhausen, and Falkner and Skan problems are considered, showing how the initial partial differential equations are reduced to ordinary differential equations. The Karman-Pohlhausen integral method is described and some approximate solutions of boundary layer problems are analyzed.

The last section of this chapter presents the natural convection, comparing it with forced convection considered in previous sections. It is noted that a free convection occurs naturally whenever there are density differences in gravitational field in contrast to the forced one, which exists due to external force. Three examples are reviewed to show the basic features of natural convective problems. Analyzing the solution for the vertical plate reveals some characteristics of natural convection that cause this type of convection to differ from the forced one. Two examples show that in case of natural convection some additional effects should be taken into account. In particular, the radiation should often be considered along with natural convection because both heat transfer rates are usually of the same order. The other effect that is significant in that case is the flow stability as, for example, in Rayleigh-Benard free convection flow between parallel horizontal plates.

Chapter 8 presents features that differentiate turbulent flow from the laminar issue. Two parts describing averaging procedure and diverse turbulence models construct this chapter. It is explained that the process of averaging parameters developed by Reynolds leads to formulating the governing equations for turbulent flow in the form similar to that for laminar flow. Presented analysis shows how the averaging procedure yields additional unknown terms in the governing equations, called the Reynolds stresses, and finally results in an unclosed system of equations. The problem of closing this system, known as a problem of closure, is solved employing the semi-empirical or statistical turbulence models.

We begin the discussion of turbulent models from the simpler algebraic models. The first Prandtl model of this type is grounded on Boussinesq relation with unknown turbulent viscosity μ_{tb} defined through the mixing-length hypothesis. The physical interpretation of both Boussinesq and Prandtl hypotheses is followed by discussion of the structure of equilibrium turbulent boundary layers, which is the basis of the modern algebraic models. The typical velocity profile in such boundary layer consists of three standard parts: the viscous sublayer where the law of the wall holds, the defect layer with Clauser's velocity law, and the overlap logarithmic region where both laws are asymptotically valid. Three modern algebraic models, Mellor-Gibson, Cebeci-Smith, and Baldwin-Lomax, are considered. The results of modeling flows in a channel, tube, in some boundary layers, and heat transfer from surface with arbitrary temperature distribution using these models show reasonable agreement with experimental data.

The remaining part of this chapter deals with the one- and two-differential equations models. These types of models grounded on the turbulence kinetic energy equation, simulate the turbulent flows much closer than algebraic models. A special section is devoted to the turbulence kinetic energy equation explaining the physical meaning of terms and the role of the Kolmogorov kinetic energy equation. Some one-equation models and results of testing these models at AFOSR conference are presented. It is noted that, according to AFOSR, the basic shortage of the one-equation models is the absence of length scale. At the same time, it is underlined that only two-equation models are complete models, which means that the solution might be obtained by the model itself without using additional experimental data.

The two most popular $k - \omega$ and $k - \varepsilon$ two-equation models are considered, and both equations defining the turbulent kinetic energy and dissipation energy rate (it serves as length scale) are discussed. The applicability of one- and two-equation models reveals that the turbulent flows with strong adverse pressure gradients, separated or reattachment flows, compressible and other complex flows may be studied with reasonable accuracy only by two-equation models since applications of more accurate new methods of direct numerical simulation are restricted at present. At the same time, the simpler algebraic models are preferable for the solution of problems with zero and benign pressure gradients.

Two parts of Chapter 9 present analytical and numerical mathematical methods frequently used in applications. To illustrate the usage of considered methods, we apply mainly problems of heat transfer in solids. Such a manner completed the set of topics important for studying the basic text since two others, laminar and turbulent fluid flow and heat transfer, are reviewed in previous chapters of Part III. The analytical methods are reviewed starting from error function. It is shown that error function satisfied the unsteady one-dimensional conduction equation and boundary conditions for infinite and semi- infinite solids and for lateral insulated thin rods. Two examples are analyzed.

The method of separation variables is considered next. Three cases are indicated when the general procedure of separation is possible for solution. Solutions of one-dimensional unsteady problems applying standard technique of Fourier series are presented. A special case when the usual Fourier series are not applicable is studied, giving an understanding of what are the eigenvalues and orthogonal eigenfunctions. The Sturm-Liouville problem is reviewed, specifying the conditions of the existing orthogonal eigenfunctions and defining the proper series. Two steady two-dimensional problems for Laplace equation with Dirichlet and mixed boundary conditions are examined as well.

The Fourier and Laplace integral transforms present the next two sections. The idea of integral transform is described, and the difference between these two widely used integral methods is explained. Four solutions for rods and rectangular sheets following this discussion show that Fourier transform is applicable to infinite domains, whereas the Laplace transform is relevant for semi-infinite positive variables domain.

The Green's method of analytical solution is described in the last section of this part of Chapter 9. The idea of this approach is close to Duhamel's method: presenting a solution of a problem with space-time dependent variables in terms of similar results for problem with constant parameters (S.1.3.1). The general formula defining the Green function is derived for the solution of a one-dimensional conduction problem.

In the second part of Chapter 9, we review shortly classical numerical methods Three sections completed this review. The first section "What method is proper" shows that there is no reason to oppose analytical and numerical methods as it becomes popular after computer advent. In the second section, we discuss the approximate methods for solving the differential equations. It is justified that these methods were developed and widely used many years before they became a basis of modern numerical methods, but before they are used for entire computation domain as analytical means. As the computers came, it became possible to apply the same approximate methods for each cell of grid vastly increasing the computing accuracy and converting these simple analytical approaches into the contemporary numerical methods. We classify numerical methods according to the types of discretization of the computation domain and analytical methods used for solution. Three methods, finite-difference (FDM), finite-element (FEM), and boundary element (BEM) methods applying uniform and irregular grids are considered.

To describe the technique of employing different analytical methods, we use the weighted residual approach. The idea of this approach is explained considering solutions of a simple conduction problem governed by a one-dimensional equation. Analysis of relevant examples clarifies the features, the merits, and lack of different methods. In particular, it is explained what is the weak solution and why the boundary element method requires data only along boundaries of computing domain, whereas the other methods demand information of the whole variable field.

The final section of Chapter 9 deals with the complications in computing flow and heat transfer characteristics. Following Patankar, we discuss some ways for overcoming problems arising in computing pressure and velocity, convection-diffusion terms, and cases of false diffusion. It is shown that the difficulty in computing flow characteristics associated with the absence of explicit equations for pressure is in fact an apparent problem because the correct pressure estimation is controlled by continuity equation. Analysis indicates that usual control volume approach fails resulting in zero pressure, and to resolve the pressure computing, the staggered control volume was developed. This procedure is described, explaining that in this case, in contrast to the usual approach, the velocity components and pressure are calculated on the control volume faces. The software SIMPLE and three modified versions of it are shortly described.

The textbook is closed with a conclusion summarizing the purpose, applicability, and prediction of the feature of the contemporary methods considered in the book.

Abram Dorfman
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About the Author

Abram S. Dorfman, Doctor of Science, Ph.D. was born in 1923 in Kiev, Ukraine in the former Soviet Union. He graduated from the Moscow Institute of Aviation in 1946, as an Engineer of Aviation Technology. From 1946 to 1947, he worked in the Central Institute of Aviation Motors (ZIAM) in Moscow. From 1947 to 1990, Dr. Dorfman studied fluid mechanics and heat transfer at the Institute of Thermophysics of Ukrainian Academy of Science in Kiev, first as a junior scientist from 1947 to 1959, then as a senior scientist from 1959 to 1978, and finally as a leading scientist from 1978 to 1990. He earned a Ph.D. with a thesis entitled "Theoretical and Experimental Investigation of Supersonic Flows in Nozzles" in 1952. In 1978, he received a Doctor of Science degree, which was the highest scientific degree in the Soviet Union, with a thesis and a book, *Heat Transfer in Flows around the Nonisothermal bodies*. From 1978 to 1990, he was associate editor of *Promyshlennaya Teploteknika,* which was published in English as *Applied Thermal Science* (Wiley). Dr. Dorfman was an adviser to graduate students for many years.

In 1990, he emigrated to the United States and continues his research as a visiting professor at the University of Michigan in Ann Arbor (since 1996). During this period, he has published several papers in leading American journals and two books, *Conjugate Problems in Convective Heat Transfer* (Taylor & Francis, 2010) and *Classical and Modern Engineering Methods in Fluid Flow and Heat Transfer,* (Momentum Press, 2013). He is listed in *Who's Who in America 2007.*

Dr. Dorfman has published more than 140 paper and four books in fluid mechanics and heat transfer in Russian (mostly) and in English. More than 50 of his papers published in Russian have been translated and are also available in English.

Nomenclature

$\mathrm{Bi} = \frac{h\Delta}{\lambda_w}$	Biot number
$\mathrm{Br} = \frac{\lambda\Delta}{\lambda_w L}\mathrm{Pr}^m\mathrm{Re}^n$	Brun number
$\mathrm{C} = \frac{u'\Delta t}{\Delta x}$	Courant number
C_1, C_2	Exponents of integral universal functions
$C_f = \frac{\tau_w}{\rho U_\infty^2}$	Friction coefficient
$2C_f/\mathrm{St}$	Reynolds analogy coefficient
c, c_p	Specific heat and specific heat at constant pressure, J/kg K
$\hat{c} = \rho c\Delta$	Thermal capacity, J/m^{2}K
D, D_h	Diameter and hydraulic diameter m
D_m	Diffusion coefficient, m^2/s
$\mathrm{Da} = \frac{k}{L^2}$	Darcy number
$\mathrm{Ec} = \frac{U^2}{c_p\theta_w}$	Eckert number
$f(\xi/x)$	Influence function of unheated zone at temperature jump
$f_q(\xi/x)$	Influence function of unheated zone at heat flux jump
$\mathrm{Fo} = \frac{\alpha t}{L^2}$	Fourier number
$\mathrm{Fr} = \frac{U}{\sqrt{gL}}$	Froude number
$\mathrm{Gr} = \frac{\beta\theta_w g L^2}{\nu^2}$	Grashof number
g_k, h_k	Coefficients of differential universal functions
g	Gravitational acceleration, m/s^2
h, h_m	Heat and mass transfer coefficients, W/ m^{2}K, W/ m^{2}s

k	Specific heat ratio or turbulence energy, m^2/s^2
K_τ, K_q	Constants of rheology laws for non-Newtonian fluids
$\mathrm{Kn} = \frac{l}{D_h}$	Knudsen number
l	Body length or mixing length, or free path, m
L	Characteristic length, m
$\mathrm{Le} = \frac{D_m}{\alpha}$	Levis number
$\mathrm{Ls} = \frac{\lambda_w h}{\rho_w^2 c_w^2 U_w^2 \Delta}$	Leidenfrost number
$\mathrm{Lu}\ \frac{\rho c}{\rho_w c_w}$	Luikov number
$\mathrm{M} = \frac{\mathrm{U}}{U_{sd}}$	Mach number
M	Moisture content, kg/kg
$n,\ s$	Exponents of rheology law for non-Newtonian fluids
$\mathrm{Nu} = \frac{hL}{\lambda},\ \mathrm{Nu} = \frac{hL^{s+1}}{K_q U^s}$	Nusselt numbers for Newtonian and non-Newtonian fluids
p	Pressure, Pa
$\mathrm{Pe} = \frac{UL}{\alpha}$	Peclet number
$\mathrm{Pr} = \frac{\nu}{\alpha},\ \mathrm{Pr} = \frac{\rho c_p U^{1-s} L^{1+s}}{K_q}$	Prandtl numbers for Newtonian and non-Newtonian fluids
$q,\ q_\mathrm{v}$	Heat flux, W/m^2 or volumetric heat source, W/m^3
r/s	Exponent of isothermal heat transfer coefficient
$\mathrm{Ra} = \frac{\beta\theta_w g L^3}{\nu\alpha}$	Rayleigh number
$\mathrm{Re} = \frac{UL}{\nu},\ \mathrm{Re}\frac{\rho U^{2-n} L^n}{K_\tau}$	Reynolds number for Newtonian and non-Newtonian fluids
$\mathrm{Sc} = \frac{\nu}{D_m}$	Schmidt number
$\mathrm{Sh} = \frac{h_m}{\rho c_p D_m}$	Sherwood number
$Sk = \frac{4\sigma T_\infty^4 L}{\lambda_\infty}$	Starks number
$\mathrm{St} = \frac{h}{\rho c_p U}$ and $\mathrm{St} = \frac{\omega L}{U}$	Stanton number and Strouhal number
$\mathrm{Ste} = \frac{c_p \Delta T}{\Lambda}$	Stephan number

$\mathrm{Stk} = \dfrac{t_p U}{D_p}$	Stokes number
t	Time, s
T	Temperature, K
$u,\ \mathrm{v}, w$	Velocity components or u, v parts of integration procedure
U	Velocity on outer edge of boundary layer
U_e	Velocity on outer edge of turbulent boundary layer
$u_\tau = \sqrt{\tau_w/\rho}$	Friction velocity
$u^+ = u/u_\tau,\ y^+ = yu_\tau/\nu$	Variables of wall law
x, y, z	Coordinates

Greek symbols

α	Thermal diffusivity, m/s^2
β	Dimensionless pressure gradient or volumetric thermal expansion coefficient, 1/K
$\chi_t = \dfrac{h}{h_*}, \chi_p = \dfrac{h_m}{h_{m*}}$	Nonisothermicity and nonisobaricity coefficients
$\chi_f = \dfrac{C_f}{C_{f*}}$	Nonisotachicity coefficient
$\delta,\ \delta_1, \delta_2$	Boundary layer thicknesses, m
$\delta,\ \delta_{ij}$	Delta function and Kronecker delta
Δ	Body or wall thickness, m
ε	Dissipation energy rate, $\mathrm{m}^2/\mathrm{s}^3$ or fraction of phase
κ	Constant determining mixing length
λ	Thermal conductivity, W/mK
Λ	Latent heat, J/kg or λ_s/λ
μ	Viscosity, kg/s m
ν	Kinematic viscosity, m^2/s
ξ	Unheated zone length, m
$\theta = T - T_\infty$,	Temperature excess, K
$\theta_w = T_w - T_\infty$	Temperature head, K
ρ	Density, kg/m^3
σ	Stefan-Boltzmann constant, $\mathrm{W}/\mathrm{m}^2\mathrm{K}^4$
τ	Shear stress, N/m^2
$\Phi,\ \varphi$	Prandtl-Mises-Görtler variables
ψ	Stream function, m^2/s
ω	Frequency or specific dissipation energy rate

Some of these symbols are also used in different ways as it is indicated in each case.

Subscripts		Superscipts	
av	Average	$+, -$	From both sides of interface
ad	Adiabatic	$+, ++$	Wall law
as	Asymptotic		
bl	Bulk	**Overscores**	
e	End or effective	$\overline{o}$, $\tilde{o}$	Dimensionless, or transformed
i	Initial; inside		
L	At $x = L$		
m	Mass average, or mean value, or moisture		
o	Outside		
p	Pressure or particle		
q	Constant heat flux		
sd	Sound		
t	Thermal		
T	Constant temperature		
tb	Turbulent		
w	Fluid-solid interface		
ξ	After jump		
∞	Far from solid		
$*$	Isothermal or special		